63-4-5

D1-82-0275

"Also available from the author"

BOEING SCIENTIFIC RESEARCH LABORATORIES

Slender, Axisymmetric Power Bodies Having Minimum Zero Lift Drag in Hypersonic Flow

Arthur H. Lusty Jr.

July 1963

D1-82-0275

BOEING SCIENTIFIC RESEARCH LABORATORIES

FLIGHT SCIENCES LABORATORY TECHNICAL REPORT NO. 77

SLENDER, AXISYMMETRIC POWER BODIES HAVING MINIMUM ZERO-LIFT DRAG IN HYPERSONIC FLOW

ARTHUR H. LUSTY JR.

JULY 1963

TABLE OF CONTENTS

SLENDER, AXISYMMETRIC POWER BODIES HAVING MINIMUM ZERO-LIFT DRAG IN HYPERSONIC FLOW

by

ARTHUR H. LUSTY, JR.(*)

SUMMARY

In this paper, the problem of finding the slender power body of revolution having minimum zero-lift drag in hypersonic flow is solved by direct methods. A constant friction coefficient is assumed, and both the Newtonian impact law and the Newton-Busemann law are employed to provide the distribution of pressure coefficients over the body. A generalized optimum condition is found in determinantal form under the assumption that any two arbitrary functions of the diameter, the length, the wetted area, the volume, and the exponent of the power body are prescribed. After these constraints are specified explicitly, particular problems are solved; it is found that, in all cases where the wetted area is not prescribed, the shape of the optimum power body is strongly dependent on the friction coefficient. For the Newtonian impact case, the power body solutions of this report are compared with the variational solutions of Refs. 1 and 2 in the range of values of the friction parameter for which the variational solution includes a single subarc only. It is found that the drag of the optimum power body approximates closely that of the variational solution body only in the cases where the diameter is one of the prescribed quantities.

(*) Staff Associate, Astrodynamics and Flight Mechanics Group, Boeing Scientific Research Laboratories.

1. INTRODUCTION

The problem of minimizing the zero-lift drag of a slender body of revolution in hypersonic flow has recently received considerable attention for the case where the pressure coefficient is assumed to satisfy the Newtonian impact law and the friction coefficient is constant (Refs. 1 and 3). While variational techniques have been employed in these references, it is the purpose of this paper to employ the ordinary theory of maxima and minima in order to restudy these problems as well as to solve the new problems arising from the use of the Newton-Busemann pressure coefficient law. This approach by direct methods is possible if the function describing the longitudinal contour of the body is prescribed except for some undetermined constants. In this connection, the class of power bodies is investigated, and the total drag (the sum of the pressure drag and the friction drag) is minimized under several constraints. A key assumption for the Newton-Busemann case is that no flow separation occurs, that is, free layers are ruled out; this is precisely the case with the power bodies as long as the exponent of the power law is larger than a certain minimum value.

2. GENERALIZED PRESSURE COEFFICIENT LAW

For a slender body of revolution, the Newtonian pressure coefficient is given by (Refs. 1 through 4)

$$C_p = 2\dot{y}^2 \tag{1}$$

where x denotes a streamwise coordinate, y a normal coordinate, and the dot sign a derivative with respect to x (Fig. 1). The Newton-Busemann pressure coefficient for this body is written as (Refs. 4 and 5)

$$C_p = 2\dot{y}^2 + y\ddot{y}$$

Either of the above relations is a particular case of the following generalized pressure coefficient law:

$$C_p = 2\left(\dot{y}^2 + \frac{\varphi}{2} y\ddot{y}\right) \tag{3}$$

where the parameter φ has the values $\varphi = 0$ for the impact case and $\varphi = 1$ for the Newton-Busemann case.

3. THE DRAG, THE WETTED AREA, AND THE VOLUME

Using the generalized pressure coefficient law of the previous section and introducing a constant friction coefficient C_f, one obtains the following expression for the drag per unit dynamic pressure of the forebody of a slender body of revolution:

$$\frac{D}{q} = 4\pi\int_0^{\ell}\left[y\dot{y}^3 + \frac{\varphi}{2}y^2\dot{y}\ddot{y}\right]dx + C_f S \tag{4}$$

where ℓ is the length of the body and S is its wetted area. Under the slender body approximation, the expression for the wetted area is written as

$$S = 2\pi\int_0^{\ell} y\ dx \tag{5}$$

while the corresponding volume is given by

$$V = \pi\int_0^{\ell} y^2\ dx \tag{6}$$

If one restricts the analysis to power bodies of the form

$$y = \frac{d}{2}\left(\frac{x}{\ell}\right)^n \tag{7}$$

where d is the diameter, the previous expressions can be integrated to yield

$$\frac{D}{q} = \frac{\pi d^4}{16\ell^2}\left[\frac{2n^3 + \varphi(n^3 - n^2)}{2n - 1}\right] + C_f S \tag{8}$$

$$S = \pi d\ell/(n + 1) \tag{9}$$

$$V = \pi d^2\ell/4(2n + 1) \tag{10}$$

provided $n > 1/2$. This limitation is necessary in both the Newtonian flow model and the Newton-Busemann flow model in order to insure that the drag contribution of the nose is finite. Incidentally, this inequality automatically insures that the pressure coefficient of the Newton-Busemann case is positive everywhere along the contour.

4. MINIMUM DRAG PROBLEM

The problem considered here is that of minimizing the right-hand side of Eq. (8) with respect to all combinations of the quantities d, ℓ, S, V, n which satisfy the fundamental constraints having the form

$$\begin{aligned} \phi_1 &\equiv S(n+1) - \pi d\ell = 0 \\ \phi_2 &\equiv 4V(2n+1) - \pi d^2\ell = 0 \end{aligned} \tag{11}$$

as well as two additional constraints having the form

$$\begin{aligned} \phi_3 &\equiv \phi_3(d, \ell, S, V, n) - \text{Const} = 0 \\ \phi_4 &\equiv \phi_4(d, \ell, S, V, n) - \text{Const} = 0 \end{aligned} \tag{12}$$

Since the number of variables is five and the number of constraints is four, the problem admits one degree of freedom. Hence, the optimizing condition can be reduced to the vanishing of only one Jacobian determinant, that of the function to be extremized and the constraining functions with respect to all the variables of the problem. This optimizing condition is given by (Refs. 6 and 7)

$$J\begin{pmatrix} D/q & \phi_1 & \phi_2 & \phi_3 & \phi_4 \\ d & \ell & S & V & n \end{pmatrix} = 0 \tag{13}$$

and its explicit form is

$$\begin{vmatrix} \frac{\partial}{\partial d}\left(\frac{D}{q}\right) & \frac{\partial}{\partial \ell}\left(\frac{D}{q}\right) & \frac{\partial}{\partial S}\left(\frac{D}{q}\right) & \frac{\partial}{\partial V}\left(\frac{D}{q}\right) & \frac{\partial}{\partial n}\left(\frac{D}{q}\right) \\ \frac{\partial \phi_1}{\partial d} & \frac{\partial \phi_1}{\partial \ell} & \frac{\partial \phi_1}{\partial S} & \frac{\partial \phi_1}{\partial V} & \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial d} & \frac{\partial \phi_2}{\partial \ell} & \frac{\partial \phi_2}{\partial S} & \frac{\partial \phi_2}{\partial V} & \frac{\partial \phi_2}{\partial n} \\ \frac{\partial \phi_3}{\partial d} & \frac{\partial \phi_3}{\partial \ell} & \frac{\partial \phi_3}{\partial S} & \frac{\partial \phi_3}{\partial V} & \frac{\partial \phi_3}{\partial n} \\ \frac{\partial \phi_4}{\partial d} & \frac{\partial \phi_4}{\partial \ell} & \frac{\partial \phi_4}{\partial S} & \frac{\partial \phi_4}{\partial V} & \frac{\partial \phi_4}{\partial n} \end{vmatrix} = 0 \qquad (14)$$

The five equations (11), (12), and (14) completely determine the set of variables d, ℓ, S, V, n which minimize the total drag per unit dynamic pressure.

5. PARTICULAR CASES

In this section, several sets of additional constraints having engineering interest are specified, and the corresponding optimum shapes and minimum values of the drag are calculated. The representation of the results is greatly facilitated if several nondimensional parameters are introduced. These parameters are the friction parameter K_f, the thickness parameter K_τ, and the drag parameter K_D. The definitions employed for these parameters depend on the particular problem and are presented in the pertinent sections.

5.1. Given Diameter and Length

If the diameter and the length are prescribed, the additional constraints are written as

$$\begin{aligned} \phi_3 &= d - \text{Const} = 0 \\ \phi_4 &= \ell - \text{Const} = 0 \end{aligned} \tag{15}$$

so that the generalized optimum condition (14) becomes

$$\begin{vmatrix} \frac{\partial}{\partial d}\left(\frac{D}{q}\right) & \frac{\partial}{\partial \ell}\left(\frac{D}{q}\right) & C_f & 0 & \frac{\partial}{\partial n}\left(\frac{D}{q}\right) \\ -\pi\ell & -\pi d & n+1 & 0 & S \\ -2\pi d\ell & -\pi d^2 & 0 & 4(2n+1) & 8V \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{vmatrix} = 0 \tag{16}$$

The expansion of this determinant yields the relationship

$$\frac{C_f S}{n + 1} = \frac{\partial}{\partial n}\left(\frac{D}{q}\right) \tag{17}$$

where

$$\frac{\partial}{\partial n}\left(\frac{D}{q}\right) = \frac{\pi d^4}{16\ell^2}\left[\frac{8n^3 - 6n^2 + \varphi(4n^3 - 5n^2 + 2n)}{(2n - 1)^2}\right] \tag{18}$$

After Eqs. (9), (17), and (18) are combined, and after the friction parameter is defined as

$$K_f = \frac{C_f \ell^3}{d^3} \tag{19}$$

the optimum condition becomes (Fig. 2)

$$K_f = \frac{n}{16}\frac{(n + 1)^2}{(2n - 1)^2}\left[2n(4n - 3) + \varphi(4n^2 - 5n + 2)\right] \tag{20}$$

Furthermore, after Eqs. (8), (9), (19), and (20) are combined and after the drag parameter is defined as(*)

(*)This drag parameter is equal to the drag coefficient C_D referred to the frontal area divided by the square of the thickness ratio $\tau = d/\ell$.

$$K_D = \frac{D}{q}\frac{4\ell^2}{\pi d^4} \tag{21}$$

the following functional relationship is obtained (Fig. 3):

$$K_D = K_D(K_f, \varphi) \tag{22}$$

5.2. Given Diameter and Wetted Area

For the problem where the diameter and the wetted area are prescribed, the additional constraints become

$$\begin{aligned} \phi_3 &\equiv d - \text{Const} = 0 \\ \phi_4 &\equiv S - \text{Const} = 0 \end{aligned} \tag{23}$$

These conditions in conjunction with the generalized optimum condition (14) yield the optimum condition for this particular problem

$$S \frac{\partial}{\partial \ell}\left(\frac{D}{q}\right) + \pi d \frac{\partial}{\partial n}\left(\frac{D}{q}\right) = 0 \tag{24}$$

After this expression is combined with Eqs. (8) and (9), the following expression is obtained for the exponent of the optimum power body:

$$6n(n - 1) + \varphi(5n^2 - 5n + 2) = 0 \tag{25}$$

Since the friction coefficient does not appear in this expression, one concludes that friction has no effect on the shape which yields the minimum drag for given diameter and wetted area. For the impact law, the exponent of the optimum power body is $n = 1$; for the Newton-Busemann law, the optimum exponent is $n = 0.761$. For the impact law, the minimum drag per unit dynamic pressure and the associated thickness ratio are given by

$$\frac{D}{q} = 0.969 \frac{d^6}{S^2} + C_f S \ , \qquad \tau = 1.571 \frac{d^2}{S} \tag{26}$$

For the Newton-Busemann pressure law, the analogous results are

$$\frac{D}{q} = 0.889 \frac{d^6}{S^2} + C_f S \ , \qquad \tau = 1.784 \frac{d^2}{S} \tag{27}$$

5.3. Given Diameter and Volume

When the diameter and the volume are prescribed, the subsidiary conditions are written as

$$\begin{aligned} \phi_3 &\equiv d - \text{Const} = 0 \\ \phi_4 &\equiv V - \text{Const} = 0 \end{aligned} \tag{28}$$

and the general optimum condition reduces to

$$8V\left[(n + 1) \frac{\partial}{\partial \ell}\left(\frac{D}{q}\right) + \pi d C_f\right] - \pi d^2\left[C_f S - (n + 1) \frac{\partial}{\partial n}\left(\frac{D}{q}\right)\right] = 0 \tag{29}$$

The length and wetted area are not prescribed for this problem so they must be determined from the fundamental constraints, which lead to

$$\ell = \frac{4V}{\pi d^2}(2n + 1) , \quad S = \frac{4V}{d}\frac{2n + 1}{n + 1} \tag{30}$$

After the friction parameter is defined as

$$K_f = C_f \frac{V^3}{d^9} \tag{31}$$

and considerable manipulations are performed, the optimum condition can be expressed as follows (Fig. 4):

$$K_f = \frac{\pi^3}{4^5}\frac{n(n + 1)^2}{(2n + 1)^3(2n - 1)^2}\left[2n(3 - 2n) - \varphi(6n^2 - 5n + 2)\right] \tag{32}$$

If the drag parameter is defined as

$$K_D = \frac{D}{q}\left(\frac{4}{\pi}\right)^3 \frac{V^2}{d^8} \tag{33}$$

a functional relationship of the form (22) can be shown to hold and is plotted in Fig. 5. Finally, if the thickness parameter is defined as

$$K_\tau = \tau \frac{V}{d^3} \tag{34}$$

the volume constraint and the optimum condition lead to the functional relationship (Fig. 6)

$$K_\tau = K_\tau(K_\ell, \varphi) \tag{35}$$

5.4. Given Diameter and Exponent

For this case, the auxiliary constraints are written as

$$\begin{aligned} \psi_3 &\equiv d - \text{Const} = 0 \\ \psi_4 &\equiv n - \text{Const} = 0 \end{aligned} \tag{36}$$

and, in conjunction with the generalized optimum condition, lead to the following relationship:

$$(n + 1) \frac{\partial}{\partial \ell}\left(\frac{D}{q}\right) + \pi d C_\ell = 0 \tag{37}$$

After the thickness parameter is defined as

$$K_\tau = \frac{\tau}{C_\ell^{1/3}} \tag{38}$$

Eq. (37) can be shown to admit the solution

$$K_\tau = 2\left[\frac{2n - 1}{(n + 1)[2n^3 + \varphi(n^3 - n^2)]}\right]^{1/3} \tag{39}$$

which is plotted in Fig. 7. Incidentally, for the particular case of a cone, the optimum thickness ratio becomes

$$\tau = [2C_f]^{1/3} \tag{40}$$

for both the impact and Newton-Busemann cases. If the drag parameter is defined as

$$K_D = \frac{D}{q} \frac{4}{\pi d^2 C_f^{\,2/3}} \tag{41}$$

and if the optimum condition (39) is combined with Eqs. (8) and (9), the minimum drag can be expressed in the following dimensionless form (Fig. 8):

$$K_D = 3\left[\frac{2n^3 + \varphi(n^3 - n^2)}{(n + 1)^2\ (2n - 1)}\right]^{1/3} \tag{42}$$

For these optimum bodies, the friction drag is two-thirds of the total drag. Furthermore, the drag parameter-exponent relationship exhibits a minimum for n = 1 in the impact case and n = 0.761 in the Newton-Busemann case (Fig. 8). These special values are those which would be obtained if the constraint on the exponent were removed and the minimal problem problem solved for given diameter only.

5.5. Given Length and Wetted Area

For the minimum drag problem with given length and wetted area, the additional constraints are written as follows:

$$\phi_3 \equiv \ell - \text{Const} = 0$$
$$\phi_4 \equiv S - \text{Const} = 0 \qquad (43)$$

If the constraints and the generalized optimum condition are combined, the exponent of the optimum power body is given by the following expression:

$$6n(4n^2 - n - 1) + \varphi(12n^3 - 13n^2 + n + 2) = 0 \qquad (44)$$

and leads to n = 0.640 for the impact law and to n = 0.606 for the Newton-Busemann law. For the impact law, the minimum drag per unit dynamic pressure and the associated thickness ratio are given by

$$\frac{D}{q} = 0.0273 \frac{S^4}{\ell^6} + C_f S , \quad \tau = 0.522 \frac{S}{\ell^2} \qquad (45)$$

For the Newton-Busemann law, the analogous results are written as

$$\frac{D}{q} = 0.0190 \frac{S^4}{\ell^6} + C_f S , \quad \tau = 0.511 \frac{S}{\ell^2} \qquad (46)$$

As in the case of given diameter and wetted area, the optimum shapes are the same as those found for the case where only the pressure drag is minimized (Refs. 2 and 5).

5.6. Given Length and Volume

For the case where the length and the volume are specified, the auxiliary constraints are written as

$$\phi_3 = \ell - \text{Const} = 0$$
$$\phi_4 = V - \text{Const} = 0 \tag{47}$$

and the generalized optimum condition can be expanded to give

$$C_f[4\pi\ell V - \pi d\ell S] + (n+1)\left[\pi d\ell \frac{\partial}{\partial n}\left(\frac{D}{q}\right) + 4V \frac{\partial}{\partial d}\left(\frac{D}{q}\right)\right] = 0 \tag{48}$$

If the fundamental constraints are used to eliminate the diameter and the wetted area and if the friction parameter is defined as

$$K_f = C_f \frac{\ell^{9/2}}{V^{3/2}} \tag{49}$$

the following relationship is obtained (Fig. 9):

$$K_f = \frac{(n+1)^2\,(2n+1)^{3/2}}{2\pi^{3/2}(2n-1)^2}\left[2n(16n^2 - 6n - 3) + \varphi(16n^3 - 18n^2 + 3n + 2)\right] \tag{50}$$

After the drag parameter and the thickness parameter are defined as

$$K_D = \frac{D}{q}\frac{\pi\ell^4}{4V^2}, \quad K_\tau = \tau \frac{\ell^{3/2}}{V^{1/2}} \tag{51}$$

they can be shown to obey functional relationships of the form (22) and (35), respectively (Figs. 10 and 11).

5.7. Given Volume and Wetted Area

For this case, the auxiliary constraints are written as

$$\begin{aligned} \psi_3 &\equiv V - \text{Const} = 0 \\ \psi_4 &\equiv S - \text{Const} = 0 \end{aligned} \tag{52}$$

and after they are combined with the generalized optimum condition, one obtains the following relationship:

$$\frac{\partial}{\partial d}\left(\frac{D}{q}\right)\left[d^2 S - 8dV\right] + \frac{\partial}{\partial \ell}\left(\frac{D}{q}\right)\left[8\ell V - 2d\ell S\right] - \frac{\partial}{\partial n}\left(\frac{D}{q}\right)\left[\pi d^2 \ell\right] = 0 \tag{53}$$

If the fundamental constraints are used to eliminate the diameter and the length from this expression, the exponent of the optimum power body can be obtained from the relation

$$6n(6n^2 - 3n - 1) + \varphi(22n^3 - 23n^2 + 5n + 2) = 0 \tag{54}$$

For the particular case of the impact law, Eq. (54) yields $n = 0.729$; for the Newton-Busemann law, the optimum exponent is $n = 0.652$. For the impact law, the minimum drag per unit dynamic pressure and the associated thickness ratio are given by

$$\frac{D}{q} = 37080 \frac{V^6}{S^8} + C_f S , \quad \tau = 58.76 \frac{V^2}{S^3} \tag{55}$$

For the Newton-Busemann law, the analogous values are

$$\frac{D}{q} = 28610 \frac{V^6}{S^8} + C_f S , \quad \tau = 59.18 \frac{V^2}{S^3} \tag{56}$$

Here again, the optimum exponent is independent of the friction coefficient since the wetted area is prescribed; thus, the optimum shapes are the same as those found when minimizing the pressure drag only (Refs. 2 and 5).

5.8. Given Volume and Exponent

For the problem in which the volume and the exponent of the power body are given, the additional constraints are written as

$$\begin{aligned} \phi_3 &\equiv V - \mathrm{Const} = 0 \\ \phi_4 &\equiv n - \mathrm{Const} = 0 \end{aligned} \tag{57}$$

and the generalized optimum condition becomes

$$d(n + 1) \frac{\partial}{\partial d}\left(\frac{D}{q}\right) - 2\ell(n + 1) \frac{\partial}{\partial \ell}\left(\frac{D}{q}\right) - \pi d\ell C_f = 0 \tag{58}$$

After the thickness parameter is defined as

$$K_\tau = \frac{\tau}{C_f^{1/3}} \tag{59}$$

and after simple manipulations are performed, Eq. (58) yields the relationship (Fig. 12)

$$K_\tau = \left[\frac{2(2n - 1)}{(n + 1)[2n^3 + \varphi(n^3 - n^2)]}\right]^{1/3} \tag{60}$$

For a cone, this relationship becomes

$$\tau = \left[\frac{C_f}{2}\right]^{1/3} \tag{61}$$

for both the impact case and the Newton-Busemann case. If the drag per unit dynamic pressure is combined with the expression for the optimum thickness and if the drag parameter is defined as

$$K_D = \frac{D}{q}\frac{1}{C_f^{8/9}V^{2/3}} \tag{62}$$

the following relationship is obtained (Fig. 13):

$$K_D = \frac{9}{2}\left(\frac{\pi}{4}\right)^{1/3}\frac{(2n + 1)^{2/3}}{n + 1}\left[\frac{(n + 1)[2n^3 + \varphi(n^3 - n^2)]}{2(2n - 1)}\right]^{1/9} \tag{63}$$

For these optimum bodies, the friction drag is eight-ninths of the total drag. Furthermore, the drag parameter-exponent relationship exhibits a minimum for n = 0.729 in the impact case and n = 0.652 in the Newton-Busemann case (Fig. 13). These special values are those which would be obtained if

the constraint on the exponent of the power law were removed and the minimal problem solved for given volume only.

6. COMPARISON OF POWER-LAW SOLUTIONS AND VARIATIONAL SOLUTIONS

Now that the optimum power bodies have been determined, it is of interest to compare their drag with that of the corresponding variational solutions where possible. In Ref. 1, the problem of the body of revolution having minimum drag in Newtonian hypersonic flow was solved for arbitrary boundary conditions. It was found that the optimum body is composed of, at most, three parts: a spike of zero thickness, a regular shape, and a cylinder depending on the boundary conditions and the friction parameter. Since the direct methods employed here have been confined to power bodies whose exponents are constant over the entire length (hence, spikes and cylinders have been excluded from this analysis), one should expect the drag of the optimum power bodies to approximate that of the variational solutions only in the case where the variational solution is composed of a regular shape only. For solutions including spikes and cylinders, the divergence between the drag of the optimum power bodies and that of the variational solution should depend on the relative length of the spike and/or the cylinder with respect to that of the regular shape, presumably increasing as this relative length increases.

With these ideas in mind, the drag of each optimum power body has been compared with that of the related variational solution of Ref. 1 for the Newtonian case and for the range of values of the friction parameter corresponding to regular shapes only. If only the diameter is given, the optimum power body is identical with the variational solution. If only the volume is prescribed, the drag of the optimum power body is 1% greater than that of the variational solution. If the diameter and length are given, variational solutions consisting of regular shapes only exist in the friction

parameter range $0 \leq K_f \leq 0.5$; in this range, the drag of the optimum power body is no more than 0.1% greater than that of the variational solutions. If the diameter and volume are prescribed, variational solutions consisting of a regular shape only exist in the friction parameter range $0 \leq K_f \leq 0.075$; in this range, the drag of the optimum power body is at most 3% greater than that of the variational solution. If the length and volume are given, variational solutions consisting of regular shapes only exist in the friction parameter range $0 \leq K_f \leq 8.03$; in this range, the drag of the optimum power body is at most 12% greater than that of the variational solution. In the cases where the wetted area is prescribed, only the pressure drag can be minimized due to the assumption of a constant friction coefficient; therefore, the absolute difference of the drag of an optimum power body and that of the corresponding variational solution is independent of the friction parameter. Hence, the relative difference decreases as the friction parameter increases. For instance, if the length and wetted areas are prescribed, the drag of the optimum power body is 13% greater than that of the variational solution if the friction parameter is zero. However, if the friction parameter were such that the friction drag was twice the pressure drag, the total drag penalty would be only 4.3%. In the case where the volume and wetted area are prescribed, the drag of the optimum power body is 14% greater than that of the variational solution if the friction parameter is zero. Finally, if the diameter and wetted area are given, the optimum power body is identical with the variational solution so that their total drags are the same. From this comparison, it appears that the drag of the optimum power body approximates well that of the variational solution as long as the diameter is given. If the diameter is not prescribed,

the approximation is generally poor—one notable exception being the case where only the volume is given.

CONCLUSIONS

In this paper, the problem of finding the slender power body of revolution having minimum zero-lift drag in hypersonic flow is solved by direct methods under the assumption that any two functions of the following quantities are given: the diameter, the length, the wetted area, the volume, and the exponent of the power law. Both the Newtonian impact law and the Newton-Busemann law are used to determine the distribution of pressure coefficients, and a constant friction coefficient is assumed. After a generalized optimum condition is found in determinantal form, particular problems are studied in detail. The analysis shows that the optimum shapes do not depend on the friction coefficient if the wetted area is given but depend on it strongly if the wetted area is free. A comparison of the solutions of this report and the variational solutions of Refs. 1 and 2 is performed for the Newtonian case and for the range of values of the friction parameter for which the variational solution consists of a single subarc only. It is found that the drag of the power-law solutions is a good approximation of the drag of the variational solutions if the diameter is given. If the diameter is not given, the approximation is generally poor — one notable exception being the case where only the volume is prescribed.

REFERENCES

1. MIELE, A. and HULL, D. G., "Slender Bodies of Revolution Having Minimum Total Drag at Hypersonic Speeds", Boeing Scientific Research Laboratories, Flight Sciences Laboratory, TR No. 70, 1963.
2. MIELE, A., "Optimum Slender Bodies of Revolution in Newtonian Flow", Boeing Scientific Research Laboratories, Flight Sciences Laboratory, TR No. 56, 1962.
3. KENNET, H., "The Effect of Skin Friction on Optimum Minimum-Drag Shapes in Hypersonic Flow", Journal of the Aerospace Sciences, Vol. 29, No. 12, 1962.
4. MIELE, A., Editor, "Extremal Problems in Aerodynamics", Academic Press, New York, 1963.
5. MIELE, A., "A Study of the Slender Body of Revolution of Minimum Drag Using the Newton-Busemann Pressure Coefficient Law", Boeing Scientific Research Laboratories, Flight Sciences Laboratory, TR No. 61, 1962.
6. MIELE, A., "Lagrange Multipliers and Quasi-Steady Flight Mechanics", Journal of the Aerospace Sciences, Vol. 26, No. 9, 1959.
7. MIELE, A., "Flight Mechanics, Vol. 2: Theory of Optimum Flight Paths", Ch. 1, Addison-Wesley Publishing Company, Reading, Massachusetts (in preparation).

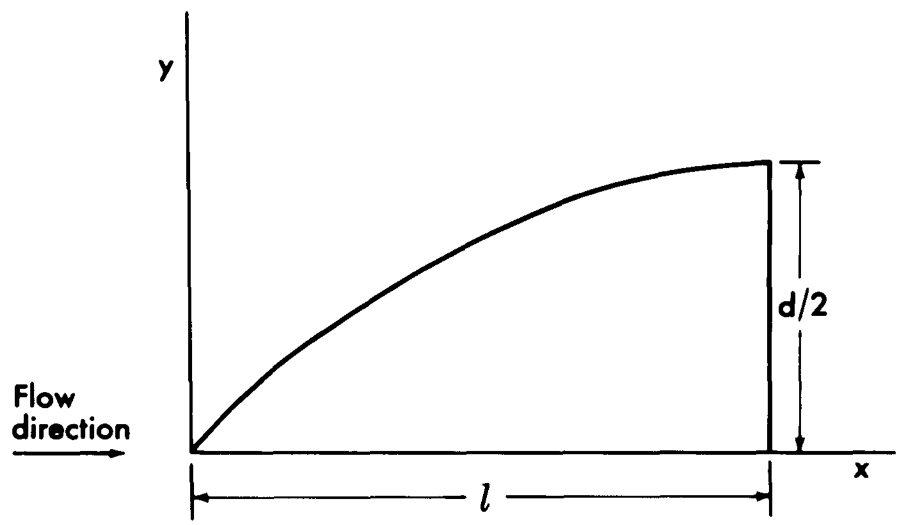

Fig. 1. Coordinate system.

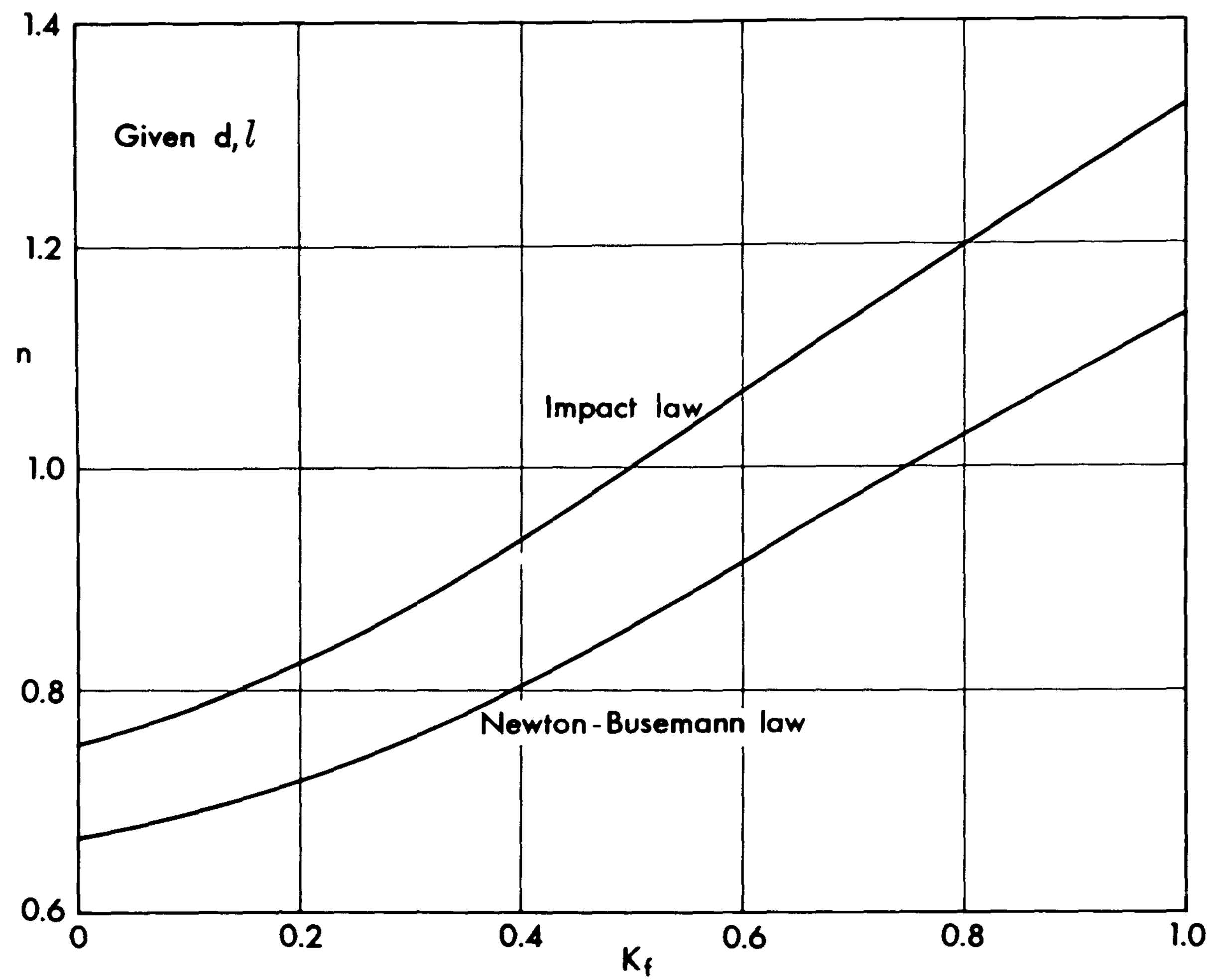

Fig. 2 Optimum exponent for given diameter and length.

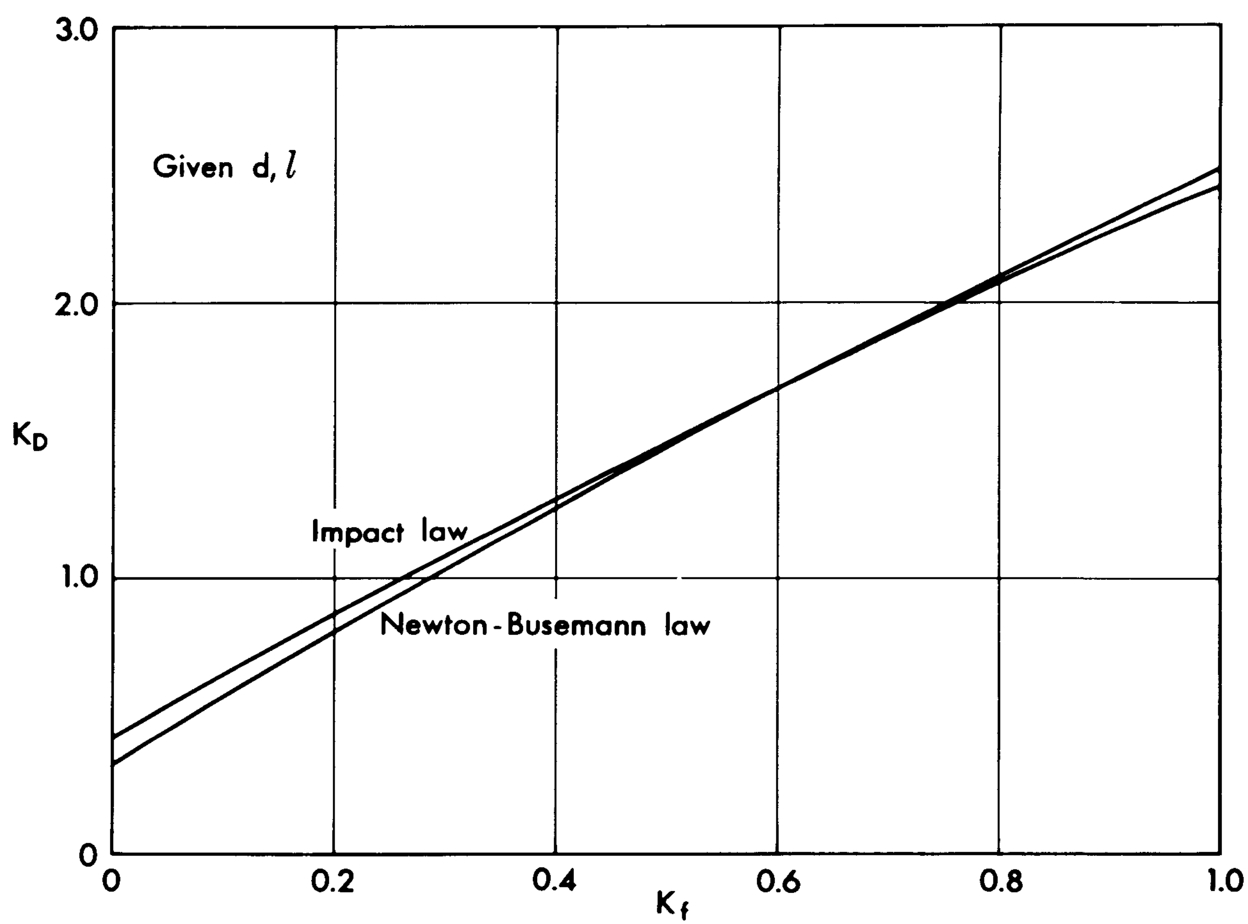

Fig. 3 Minimum drag factor for given diameter and length.

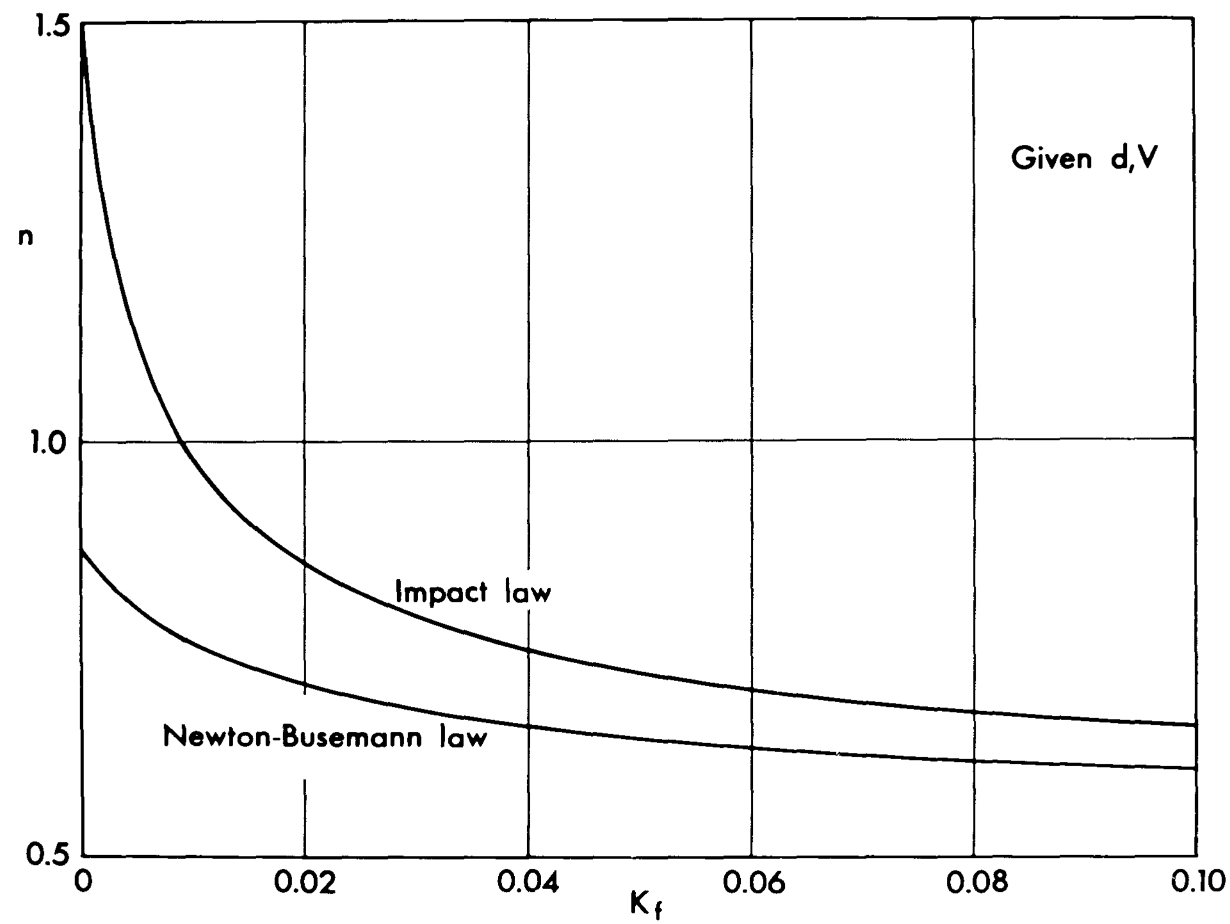

Fig. 4 Optimum exponent for given diameter and volume.

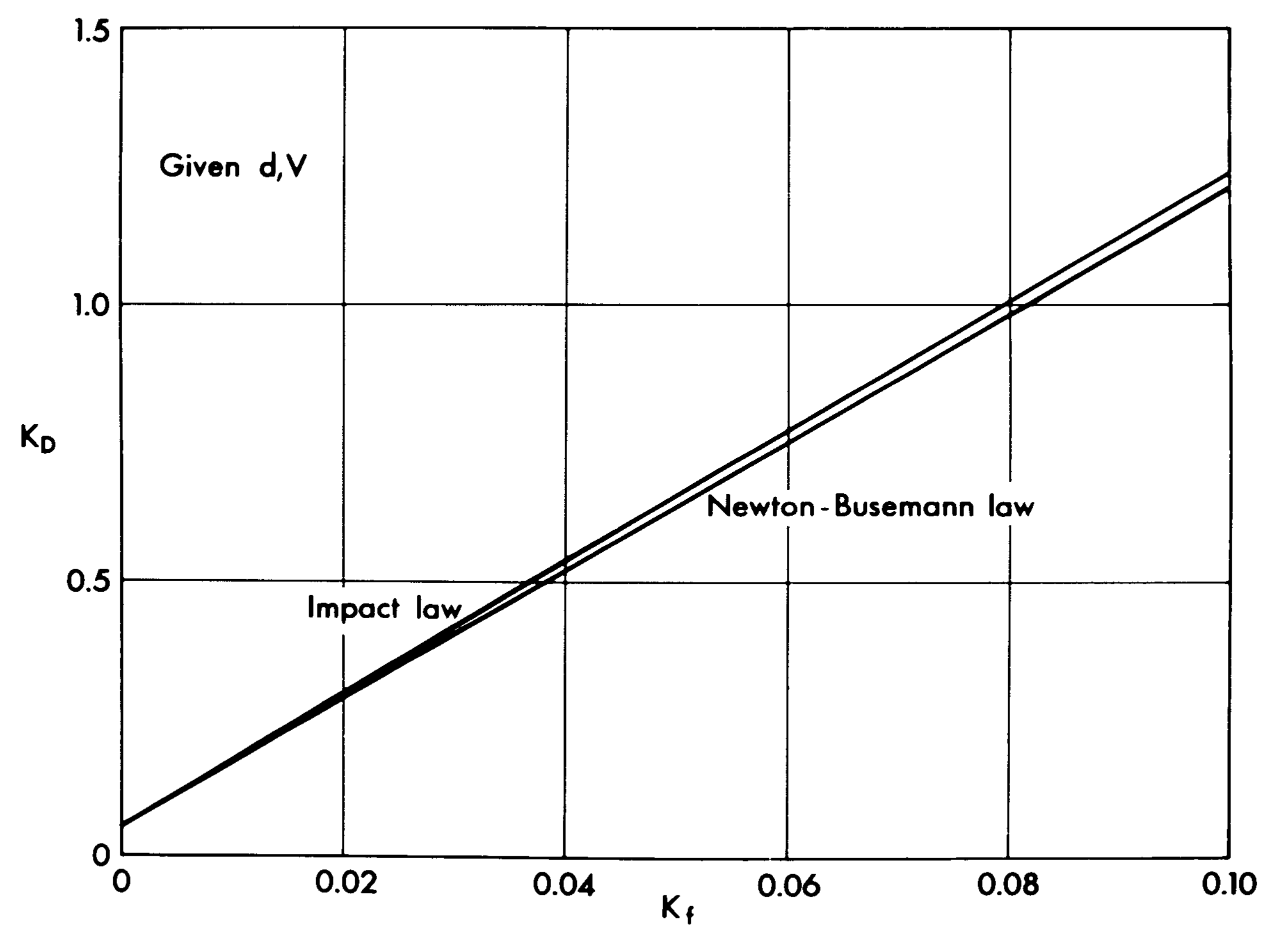

Fig. 5 Minimum drag factor for given diameter and volume.

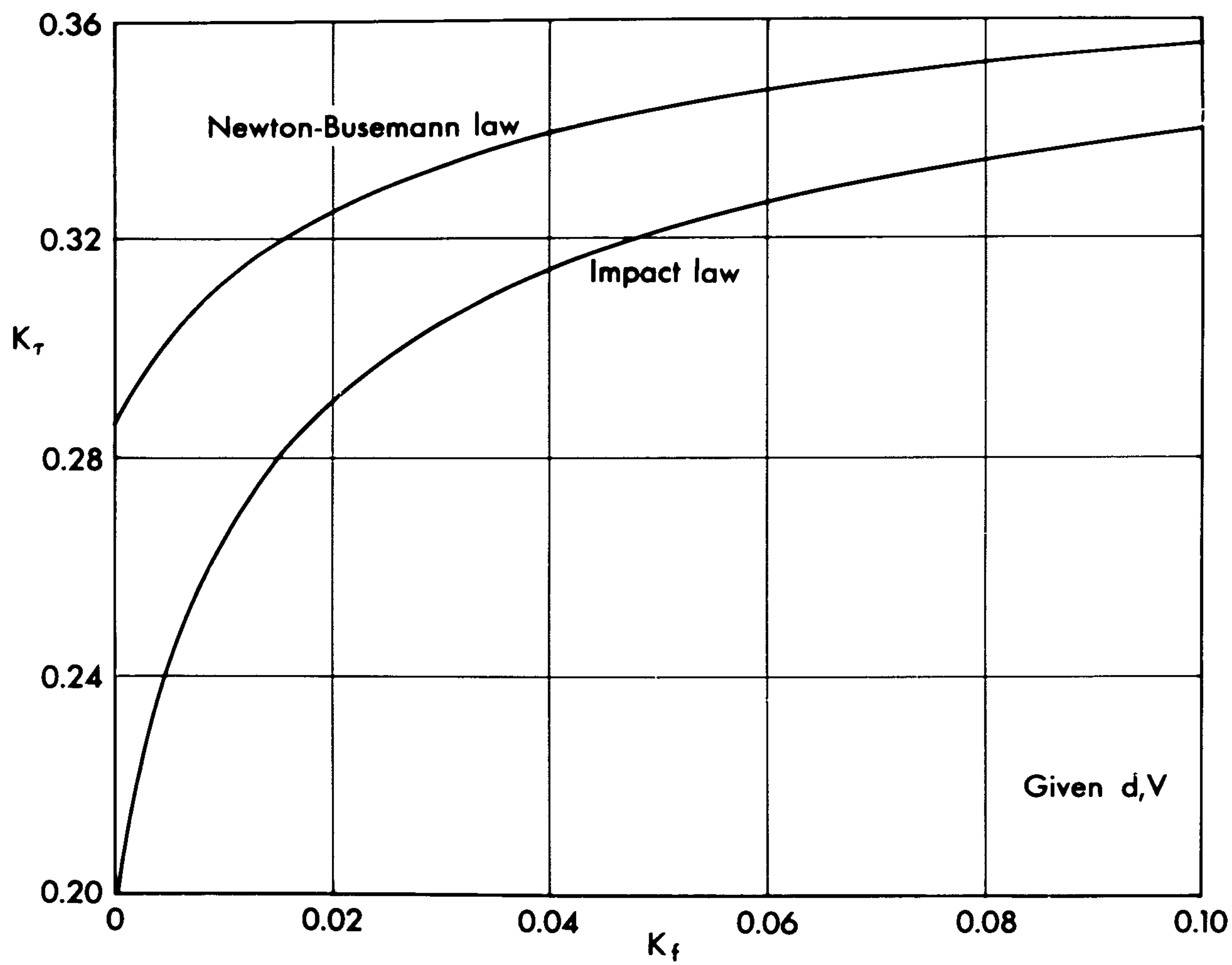

Fig. 6 Optimum thickness parameter for given diameter and volume.

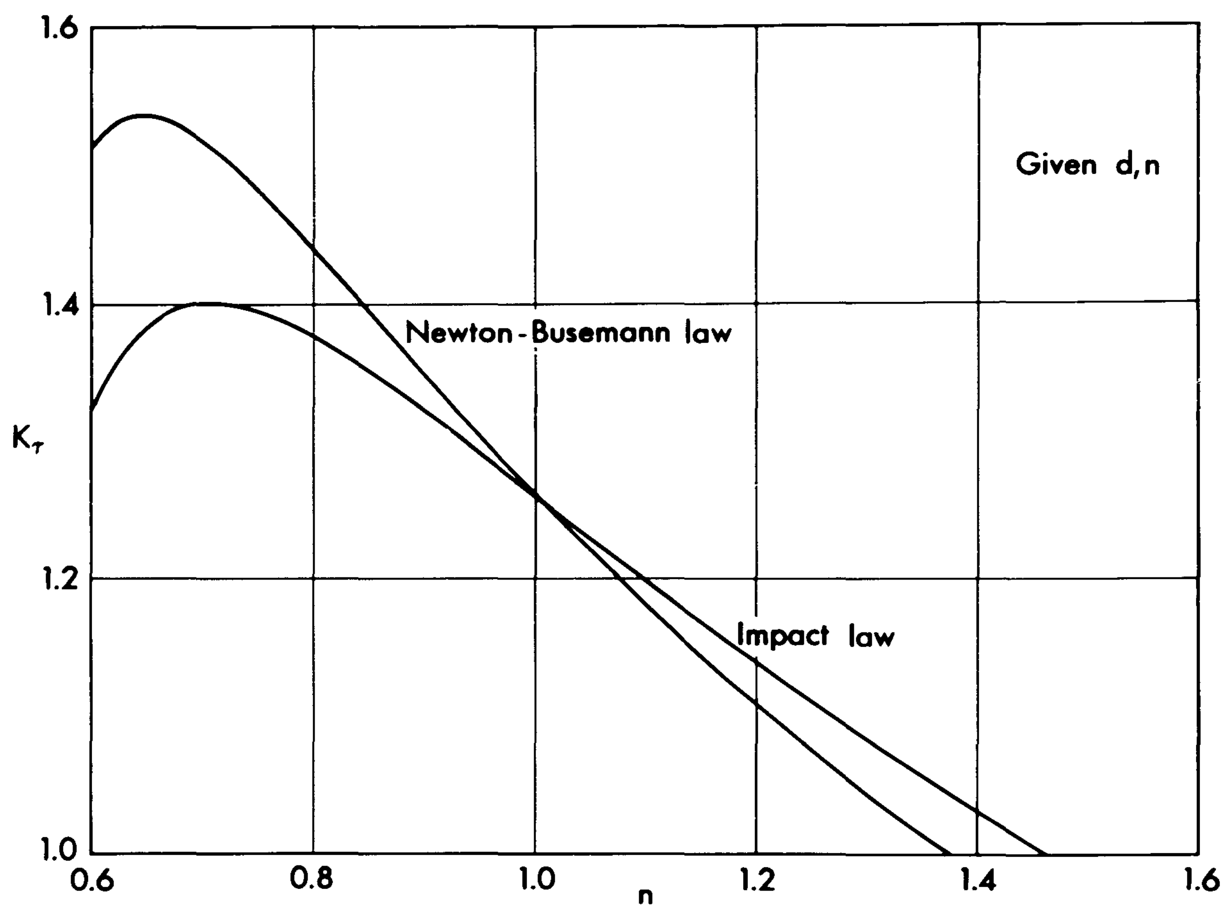

Fig. 7 Optimum thickness parameter for given diameter and exponent.

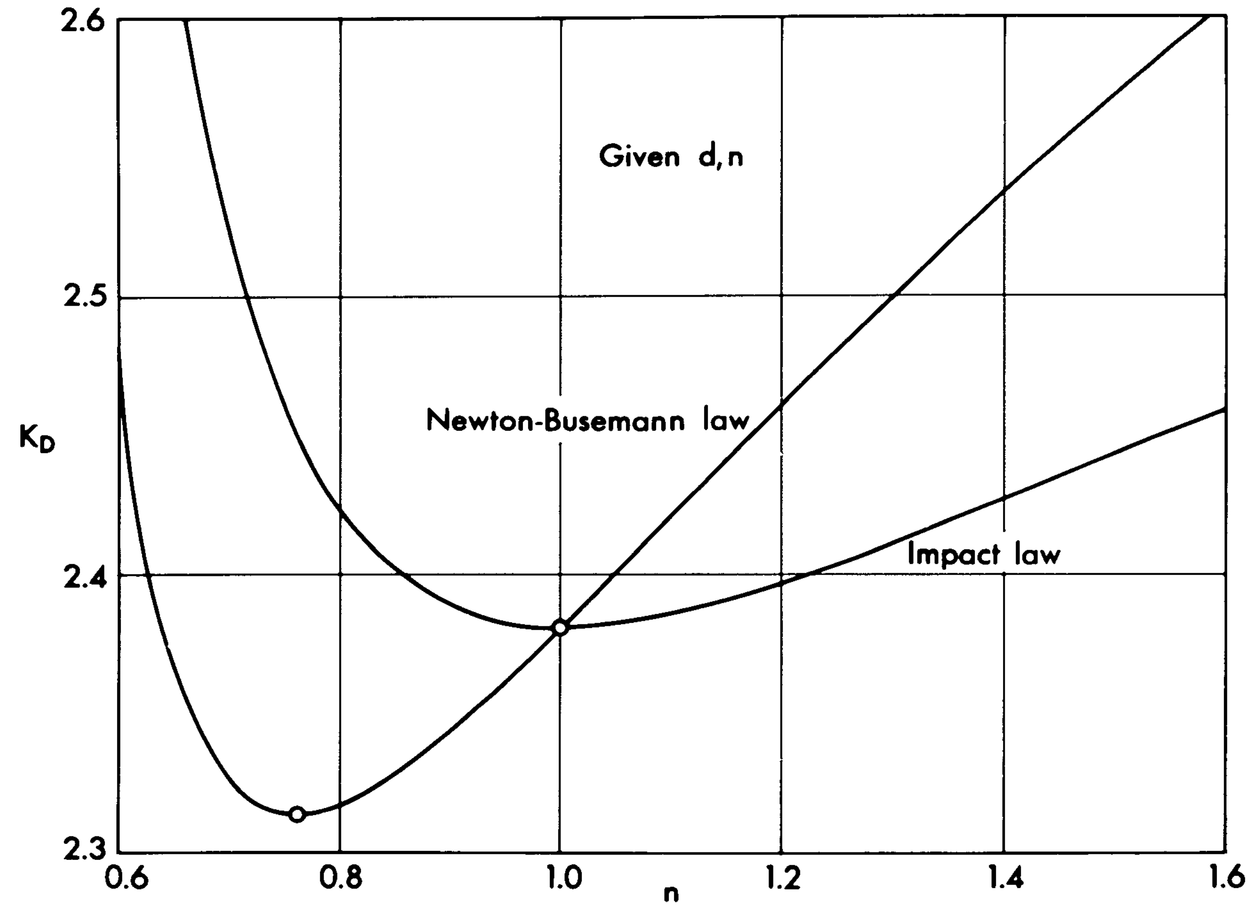

Fig. 8 Minimum drag factor for given diameter and exponent.

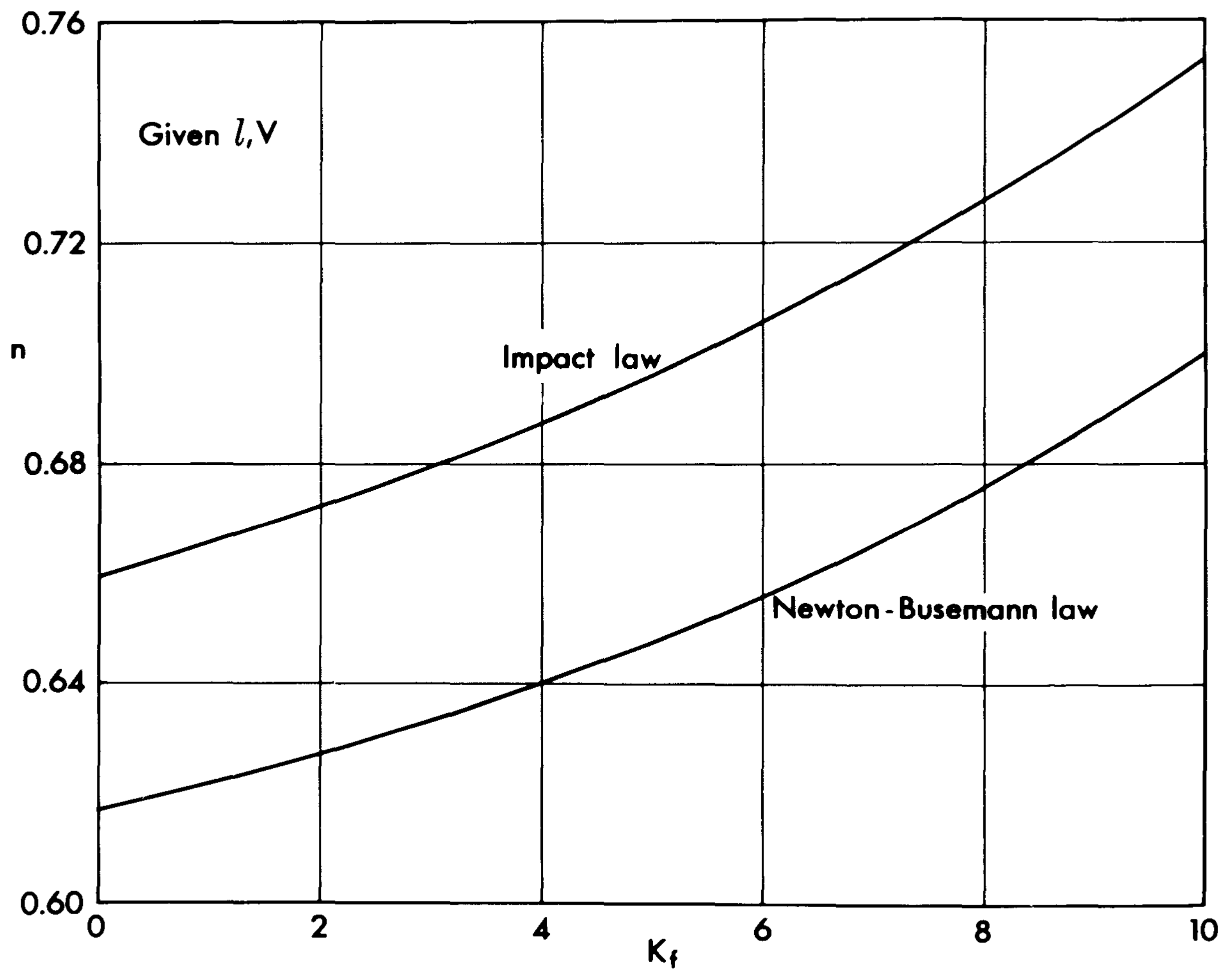

Fig. 9 Optimum exponent for given length and volume.

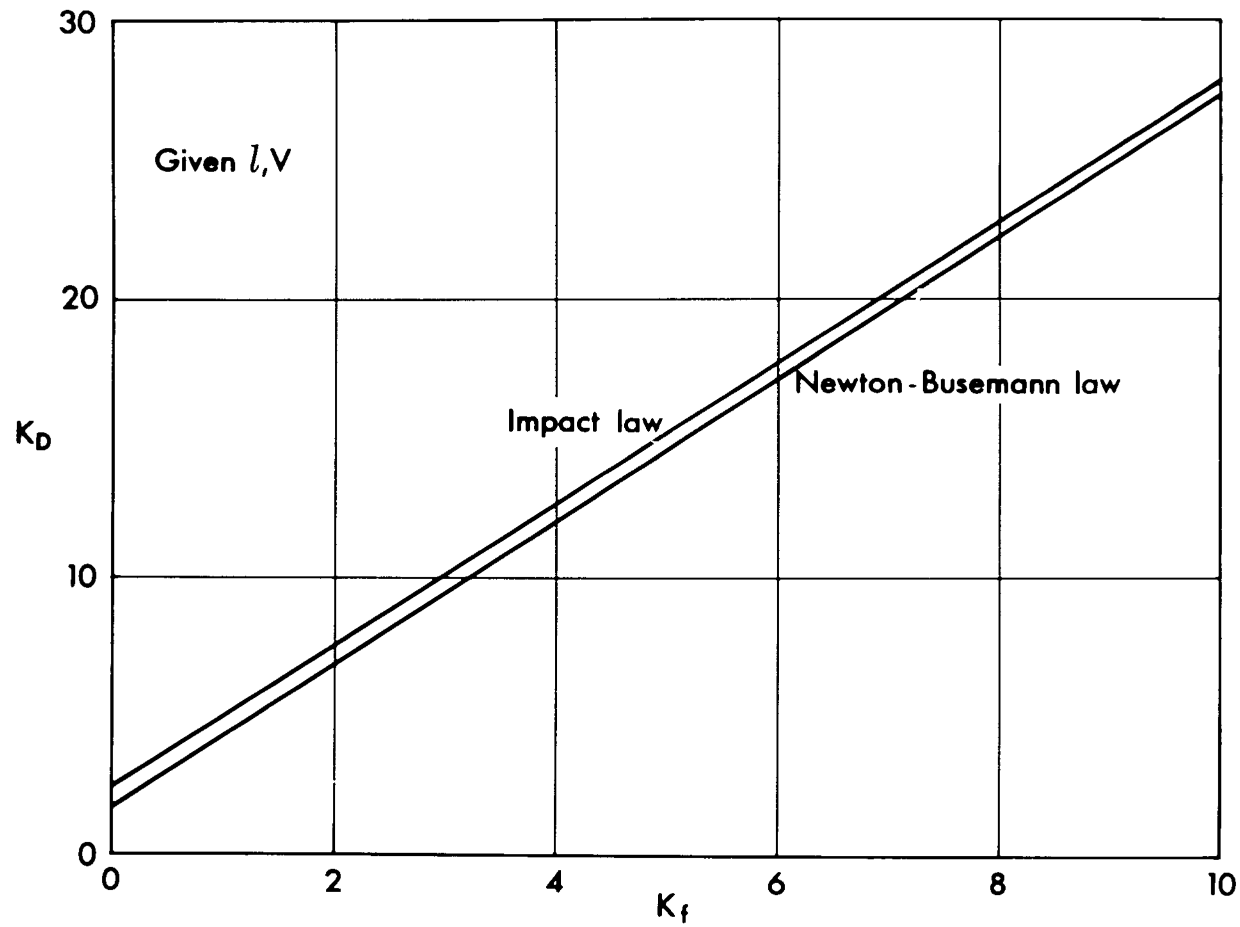

Fig. 10 Minimum drag factor for given length and volume.

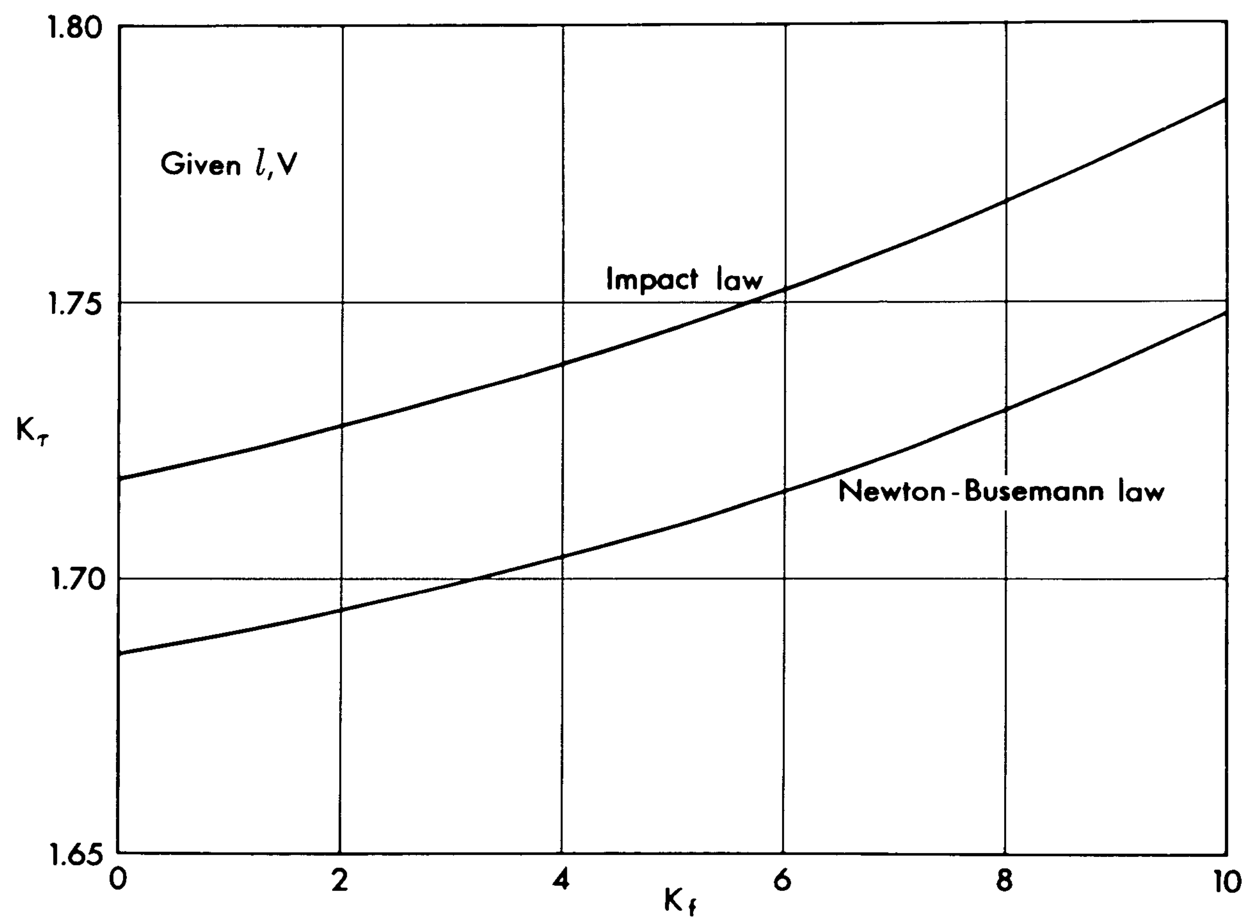

Fig. 11 Optimum thickness parameter for given length and volume.

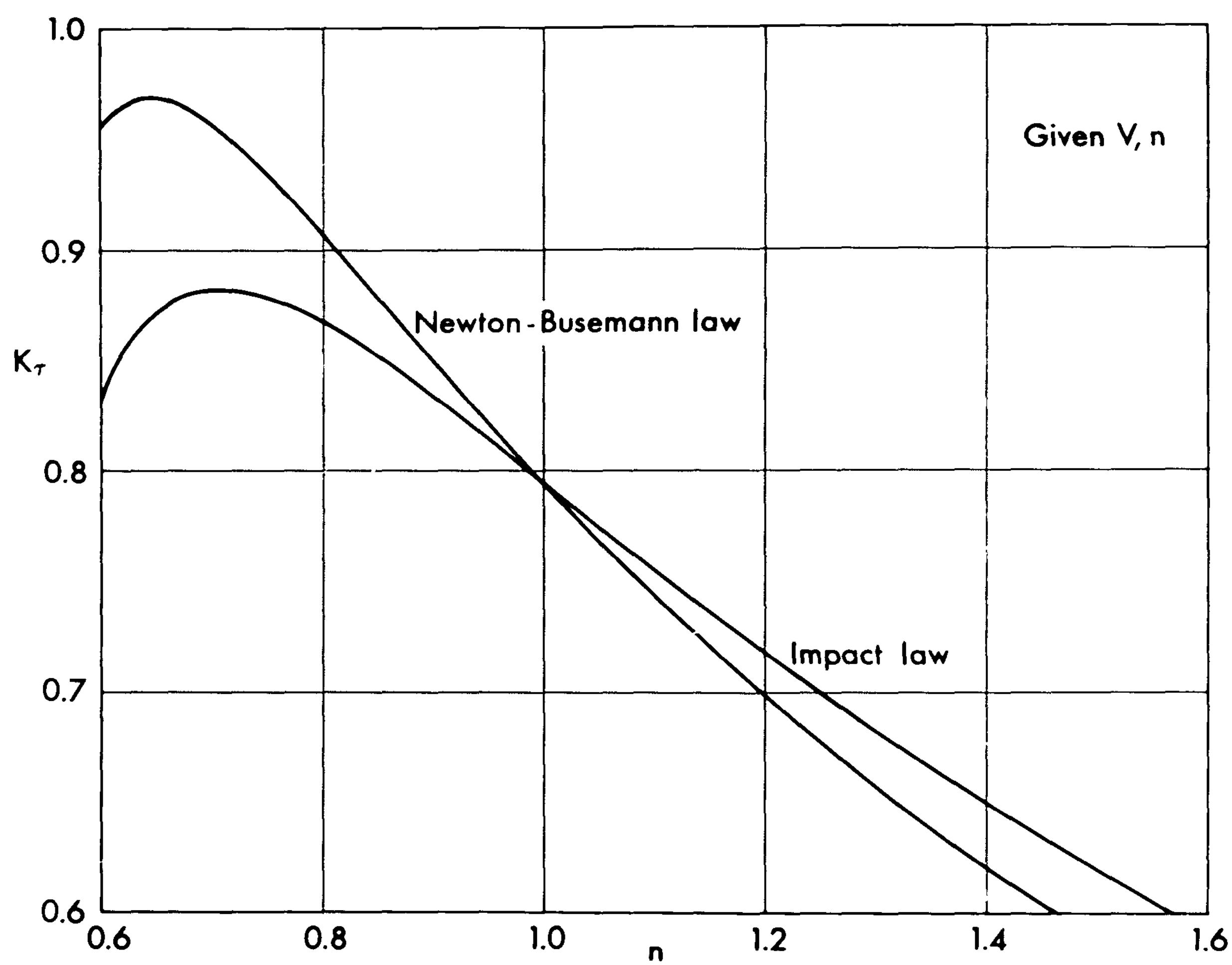

Fig. 12 Optimum thickness parameter for given volume and exponent.

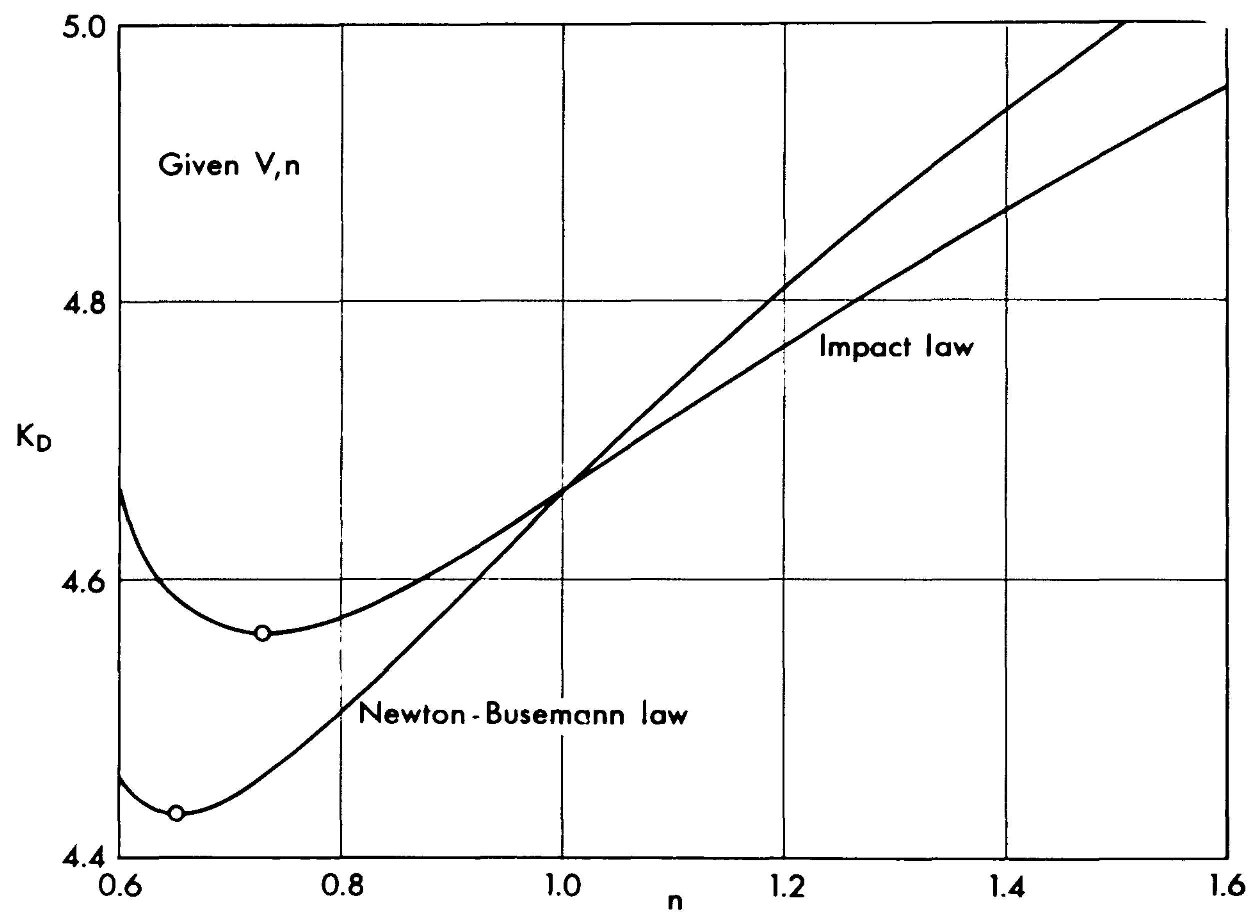

Fig. 13. Minimum drag factor for given volume and exponent.